$$= \arg \min_{g(n)\in\mathcal{L}} W_1 \left(g^t\Theta(\theta_{1_1},\theta_{1_2})g - s^t\Theta(\theta_{1_1},\theta_{1_2})g - g^t\Theta(\theta_{1_1},\theta_{1_2})s + s^t\Theta(\theta_{1_1},\theta_{1_2})s \right) + \sum_{l=2}^{Q} W_l g^t\Theta(\theta_{l_1},\theta_{l_2})g \tag{25}$$

$$= \arg \min_{g(n)\in\mathcal{L}} \sum_{l=1}^{Q} W_l g^t\Theta(\theta_{l_1},\theta_{l_2})g - W_1 \left(s^t\Theta(\theta_{l_1},\theta_{l_2})g + g^t\Theta(\theta_{l_1},\theta_{l_2})s - s^t\Theta(\theta_{l_1},\theta_{l_2})s \right) \tag{26}$$

In the penultimate line, equation (8) was substituted. To shorten writing, the two quantities $\theta_{1_1} = k\frac{2\pi}{M}$ and $\theta_{1_2} = (k+1)\frac{2\pi}{M}$ were introduced. The pulse response of the nominal transmission function is combined in the column vector *s*,

$$[s]_n = -\frac{1}{2}\left(e^{j\frac{\Delta\phi}{2}} h_{k-1}(n) + e^{-j\frac{\Delta\phi}{2}} h_{k+2}(n) \right) \quad \text{for} \quad n = 0, 1, \ldots, M-1 \ . \tag{27}$$

The direct minimization via *g(n)*, as it is executed in (26) does not make sense for two reasons: first, minimization has to be done over a large number of parameters (M coefficients) and second, optimization has to take place under the boundary condition $g(n) \in L$. It makes more sense to minimize, starting from the coefficient vector *c* (19): first, the number of the parameters to be optimized is reduced and second, minimization may occur without secondary conditions as the statement (19) already takes the secondary condition $g(n) \in L$ into consideration. Substitution of (19) in (26) yields the following:

$$c_{opt} = \arg\min_{c} \sum_{l=1}^{Q} W_l c^t H^t \Theta(\theta_{l_1}, \theta_{l_2}) H c - \tag{28}$$

$$W_1 \left(s^t \Theta(\theta_{l_1}, \theta_{l_2}) H c + c^t H^t \Theta(\theta_{l_1}, \theta_{l_2}) s - s^t \Theta(\theta_{l_1}, \theta_{l_2}) s \right) \tag{29}$$

The solution to this optimization problem reads

$$c_{opt} = H^{-1} \left(H^t \sum_{l=1}^{Q} W_l \Theta(\theta_{l_1}, \theta_{l_2}) \right)^{-1} W_1 H^t \Theta(\theta_{l_1}, \theta_{l_2}) s \, . \tag{30}$$

Substitution in (19) yields the pulse response of the compensation pulse looked for.

$$g = H c_{opt} = \left(H^t \sum_{l=1}^{Q} W_l \Theta(\theta_{l_1}, \theta_{l_2}) \right)^{-1} W_1 H^t \Theta(\theta_{l_1}, \theta_{l_2}) s \tag{31}$$

Simulations showed that this pulse only insufficiently meets the required properties. For this reason, the compensation pulse must be allowed to have a length superior to M. At this point it also makes sense to relinquish the restriction of the distortion-free channel.

It is assumed that the channel possesses a memory length of maximum P, the pulse response of the channel accordingly having a length of maximum $P + 1$ taps. For DMT systems, a cyclical

prefix is used which substantially simplifies distortion in the receiver.

With the cyclical prefix, the last P symbols of a data block are sent first at the beginning of a block, compare ***FIG. 15.*** Considering the transmission of one single pulse only, it may be written for the transmission sequence *y(n)*

$$y(n) = \begin{cases} a_{M-P+n} & \text{für} \quad n = 0, 1, \ldots, P-1 \\ a_{n-P} & \text{für} \quad n = P, P+1, \ldots, N+P-1 \end{cases} \tag{32}$$

Substitute the IDFT for a_n yields

$$y(n) = \begin{cases} \frac{1}{M} \sum\limits_{k=0}^{M-1} A_k e^{j\frac{2\pi}{M}k(M-P+n)} = \frac{1}{M} \sum\limits_{k=0}^{M-1} A_k e^{j\frac{2\pi}{M}k(n-P)} & \text{für} \quad n = 0, 1, \ldots, P-1 \\ \frac{1}{M} \sum\limits_{k=0}^{M-1} A_k e^{j\frac{2\pi}{M}k(n-P)} & \text{für} \quad n = P, P+1, \ldots, N+P-1 . \end{cases} \tag{33}$$

As can be seen, the case discrimination is abolished on account of the M periodicity of the IDFT, the transmission sequence reads

$$y(n) = \frac{1}{M} \sum_{k=0}^{M-1} A_k e^{j\frac{2\pi}{M}k(n-P)} \quad \text{for} \quad n = 0, 1, \ldots, N+P-1 . \tag{34}$$